

Calculators and mobile phones are not allowed.

Answer all of the following questions.

Each (sub)question is worth 5 points.

1. Find the limit

$$\lim_{x \rightarrow 1^+} (\ln x)^{\ln x}.$$

2. Evaluate the integrals

(a) $\int \sqrt{\sec x} \sin^3 x \, dx,$

(b) $\int 2x \ln(x^3 + x) \, dx,$

(c) $\int \frac{2 \, dx}{1 + x + x^2 + x^3},$

(d) $\int \frac{dx}{8 - 4 \sin x + 7 \cos x}.$

3. Determine if the following improper integral converges or diverges, and if it converges, find its value

$$\int_0^{\infty} \frac{dx}{e^x + e^{-x}}$$

4. Find the arc length of the parametric curve

$$x = 2e^t + 3$$

$$y = 2t - 5 \quad (\ln \sqrt{3} \leq t \leq \ln \sqrt{8}).$$

5. Find the area of the region that is inside the graph of the curve $r = 6 \sin \theta$ and outside the graph of $r = 3$.